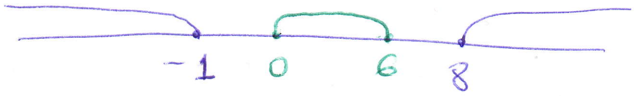
Логаритмични неравенства

1) $\log_{0,6}(6x - x^2) > \log_{0,6}(-8-x) \Leftrightarrow$

$$\begin{cases} 6x - x^2 < -8 - x \\ 6x - x^2 > 0 \end{cases} \Leftrightarrow \begin{cases} x^2 - 7x - 8 > 0 \\ x^2 - 6x < 0 \end{cases} \Leftrightarrow$$

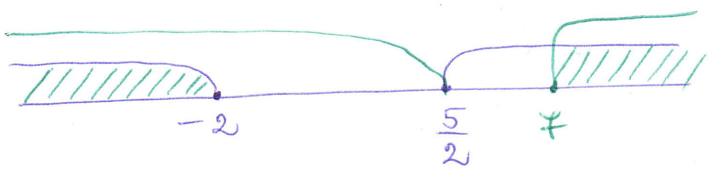
$$\Leftrightarrow \begin{cases} (x-8)(x+1) > 0 \\ x(x-6) < 0 \end{cases}$$



Няма решения

2) $\log_3 \frac{x-7}{2x-5} < 0 \Leftrightarrow \log_3 \frac{x-7}{2x-5} < \log_3 1 \Leftrightarrow$

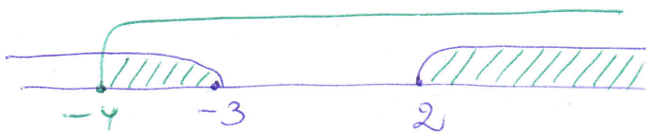
$$\begin{cases} \frac{x-7}{2x-5} < 1 \\ \frac{x-7}{2x-5} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{x+2}{2x-5} > 0 \\ \frac{x-7}{2x-5} > 0 \end{cases}$$



$x \in (-\infty; -2) \cup (7; +\infty)$.

3) $\log_{\frac{1}{3}}(x+4) > \log_{\frac{1}{3}}(x^2 + 2x - 2) \Leftrightarrow$

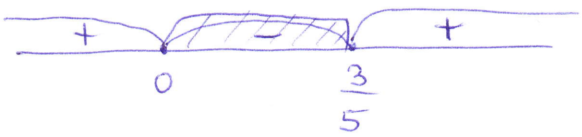
$$\begin{cases} x + 4 < x^2 + 2x - 2 \\ x + 4 > 0 \end{cases} \Leftrightarrow \begin{cases} x^2 + x - 6 > 0 \Leftrightarrow (x+3)(x-2) > 0 \\ x > -4 \end{cases}$$



$x \in (-4; -3) \cup (2; +\infty)$

4) $\log_3 \dfrac{2-3x}{x} \ge -1 \iff \log_3 \dfrac{2-3x}{x} \ge \log_3 \dfrac{1}{3} \iff$

$\dfrac{2-3x}{x} \ge \dfrac{1}{3} \iff \dfrac{6-10x}{3x} \ge 0 \iff \dfrac{5x-3}{x} \le 0$



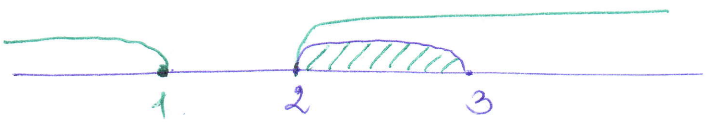
$x \in \left(0; \frac{3}{5}\right]$

5) $1 + \log_2(x-2) > \log_2(x^2-3x+2) \iff$

$\log_2 2(x-2) > \log_2(x^2-3x+2) \iff$

$\begin{cases} 2(x-2) > x^2-3x+2 \\ x^2-3x+2 > 0 \end{cases} \iff \begin{cases} x^2-5x+6 < 0 \\ x^2-3x+2 > 0 \end{cases} \iff$

$\iff \begin{cases} (x-3)(x-2) < 0 \\ (x-1)(x-2) > 0 \end{cases}$



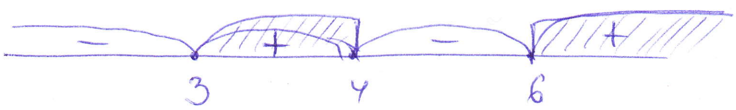
$x \in (2; 3)$

6) $\log_{\frac{1}{2}}\left(\log_8 \dfrac{x^2-2x}{x-3}\right) \le 0 \iff \log_{\frac{1}{2}}\left(\log_8 \dfrac{x^2-2x}{x-3}\right) \le \log_{\frac{1}{2}} 1$

$\iff \log_8 \dfrac{x^2-2x}{x-3} \ge 1 \iff \log_8 \dfrac{x^2-2x}{x-3} \ge \log_8 8 \iff$

$\dfrac{x^2-2x}{x-3} \ge 8 \iff \dfrac{x^2-2x}{x-3} - 8 \ge 0 \iff \dfrac{x^2-2x-8x+24}{x-3} \ge 0$

$\iff \dfrac{x^2-10x+24}{x-3} \ge 0 \iff \dfrac{(x-4)(x-6)}{x-3} \ge 0$

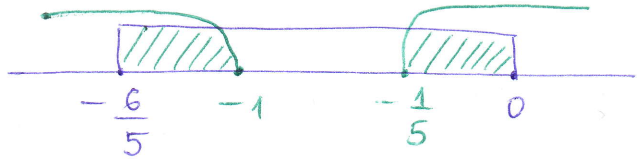


$x \in (3; 4] \cup [6; +\infty)$

7) $\log_3(5x^2+6x+1) \le 0 \iff \log_3(5x^2+6x+1) \le \log_3 1$

$$\iff \begin{cases} 5x^2+6x+1 \le 1 \\ 5x^2+6x+1 > 0 \end{cases} \iff \begin{cases} 5x^2+6x \le 0 \\ 5x^2+6x+1 > 0 \end{cases} \iff$$

$$\iff \begin{cases} x(5x+6) \le 0 \\ 5(x+1)\left(x+\frac{1}{5}\right) > 0 \end{cases}$$



$$x \in \left[-\frac{6}{5}; -1\right) \cup \left(-\frac{1}{5}; 0\right]$$

8) $\log_3(x+2) + \log_3(x-4) - 1 \le 0 \iff$

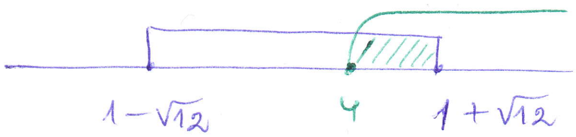
$\log_3(x+2) + \log_3(x-4) \le \log_3 3 \iff$

$$\begin{cases} x+2>0 \\ x-4>0 \\ \log_3(x+2)(x-4) \le \log_3 3 \end{cases} \iff \begin{cases} x>4 \\ (x+2)(x-4) \le 3 \end{cases} \iff$$

$$\begin{cases} x>4 \\ x^2-2x-11 \le 0 \end{cases}$$

$x^2-2x-11=0$

$x_{1,2} = 1 \pm \sqrt{12}$



$$x \in (4; 1+\sqrt{12}]$$

9) $1 + \log_{\frac{1}{4}}\left(\log_3(4-x)\right) > 0 \iff \log_{\frac{1}{4}}\left(\log_3(4-x)\right) > \log_{\frac{1}{4}} 4$

$$\iff \begin{cases} \log_3(4-x) < 4 \\ \log_3(4-x) > 0 \end{cases} \iff \begin{cases} \log_3(4-x) < \log_3 81 \\ \log_3(4-x) > \log_3 1 \end{cases} \iff$$

$$\iff \begin{cases} 4-x < 81 \\ 4-x > 1 \end{cases} \iff \begin{cases} x > -77 \\ x < 3 \end{cases} \iff x \in (-77; 3)$$

10) $\log_2 (x-3)(x+2) + \log_{\frac{1}{2}} (x+2)(x-6) \le 2 \iff$

$$\log_2 (x-3)(x+2) + \frac{\log_2 (x+2)(x-6)}{\log_2 \frac{1}{2}} \le 2 \iff$$

$$\log_2 (x-3)(x+2) - \log_2 (x+2)(x-6) \le 2 \iff$$

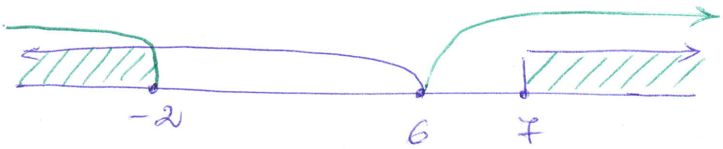
$$\left| \begin{array}{l} \left.\begin{array}{l} (x-3)(x+2) > 0 \\ (x-6)(x+2) > 0 \end{array}\right\} \iff x < -2 \text{ или } x > 6 \\ \log_2 \dfrac{(x-3)(x+2)}{(x-6)(x+2)} \le \log_2 4 \iff \log_2 \dfrac{x-3}{x-6} \le \log_2 4 \end{array} \right. \iff$$

$$\iff \left| \begin{array}{l} x < -2 \text{ или } x > 6 \Rightarrow \dfrac{x-3}{x-6} > 0 \\ \dfrac{x-3}{x-6} \le 4 \end{array} \right.$$

затова не е необходимо да го записваме в с-мата.

$$\iff \left| \begin{array}{l} x < -2 \text{ или } x > 6 \\ \dfrac{x-3-4x+24}{x-6} \le 0 \end{array} \right. \iff$$

$$\iff \left| \begin{array}{l} x < -2 \text{ или } x > 6 \\ \dfrac{21-3x}{x-6} \le 0 \iff \dfrac{x-7}{x-6} \ge 0 \end{array} \right.$$

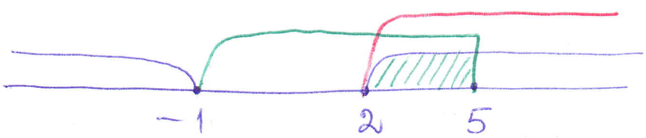


Отговор $x \in (-\infty; -2) \cup [7; +\infty)$

11) $2\log_{\frac{1}{2}} (x-2) - \log_{\frac{1}{2}} (x^2-x-2) \ge 1 \iff \left| \begin{array}{l} x > 2 \\ x^2-x-2 > 0 \\ \log_{\frac{1}{2}} \dfrac{(x-2)^2}{x^2-x-2} \ge \log_{\frac{1}{2}} \dfrac{1}{2} \end{array} \right.$

$$\iff \left| \begin{array}{l} x > 2 \\ (x-2)(x+1) > 0 \\ \dfrac{(x-2)^2}{x^2-x-2} \le \dfrac{1}{2} \end{array} \right. \iff \left| \begin{array}{l} x > 2 \\ (x-2)(x+1) > 0 \\ \dfrac{2x^2-8x+8-x^2+x+2}{2(x-2)(x+1)} \le 0 \end{array} \right. \iff$$

$$\iff \left| \begin{array}{l} x > 2 \\ (x-2)(x+1) > 0 \\ \dfrac{x^2-7x+10}{(x-2)(x+1)} \le 0 \end{array} \right. \iff \left| \begin{array}{l} x > 2 \\ (x-2)(x+1) > 0 \\ \dfrac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}(x+1)} \le 0 \end{array} \right.$$



$x \in (2; 5]$

12) $\log_{x^2+4} 8 < 1$

поради $x^2 \geq 0$, то $x^2+4 \geq 4 \Rightarrow \log_{x^2+4} 8 < \log_{x^2+4}(x^2+4)$

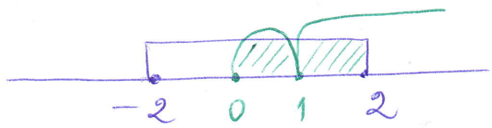
$\Leftrightarrow 8 < x^2+4 \Leftrightarrow x^2-4 > 0 \Leftrightarrow \underline{x \in (-\infty; -2) \cup (2; +\infty)}.$

13) $\frac{1}{\log_x 2} - \log_2 \frac{1}{x} \leq 2 \Leftrightarrow$

$\Rightarrow$

$$\begin{cases} x > 0 \\ x \neq 1 \\ \log_2 x - \log_2 \frac{1}{x} \leq 2 \end{cases} \Leftrightarrow \log_2 x^2 \leq \log_2 4 \Leftrightarrow x^2 \leq 4$$

$x \in (0;1) \cup (1;2]$



14) $\log_{x-1}(x+2) \leq 0 \Leftrightarrow \log_{x-1}(x+2) \leq \log_{x-1} 1 \Leftrightarrow$

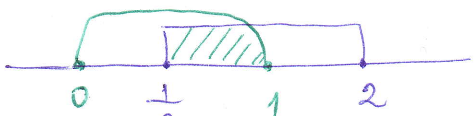
$$\begin{cases} x-1 > 1 \\ x+2 \leq 1 \\ x+2 > 0 \end{cases} \cup \begin{cases} 0 < x-1 < 1 \\ x+2 \geq 1 \end{cases} \Leftrightarrow$$

$$\begin{cases} x > 2 \\ x \leq -1 \\ x > -2 \end{cases} \not\leftrightarrow \cup \begin{cases} 1 < x < 2 \\ x \geq -1 \end{cases} \Leftrightarrow \underline{x \in (1; 2)}$$

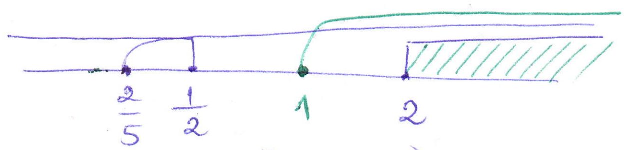
15) $\log_{\frac{1}{x}}\left(\frac{5x}{2} - 1\right) \geq -2 \Leftrightarrow \log_{\frac{1}{x}}\left(\frac{5x}{2} - 1\right) \geq \log_{\frac{1}{x}} x^2 \Leftrightarrow$

$$\begin{cases} \frac{1}{x} > 1 \\ \frac{5x}{2} - 1 \geq x^2 \end{cases} \cup \begin{cases} 0 < \frac{1}{x} < 1 \\ \frac{5x}{2} - 1 \leq x^2 \\ \frac{5x}{2} - 1 > 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \frac{x-1}{x} < 0 \Leftrightarrow x \in (0;1) \\ 2x^2 - 5x + 2 \leq 0 \end{cases} \cup \begin{cases} x > 0, \ \frac{x-1}{x} > 0 \Leftrightarrow x > 1 \\ 2x^2 - 5x + 2 \geq 0 \\ 5x - 2 > 0 \Leftrightarrow x > \frac{2}{5} \end{cases}$$



$x \in [\frac{1}{2}; 1)$



$x \in [2; +\infty)$

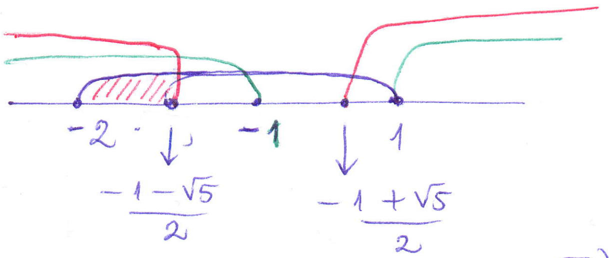
Одговор: $x \in [\frac{1}{2}; 1) \cup [2; +\infty)$

16) $\log_{x^2}(x^2+x-1)<0 \iff \log_{x^2}(x^2+x-1)<\log_{x^2}1 \iff$

$$\left|\begin{array}{l} x^2>1 \\ x^2+x-1<1 \\ x^2+x-1>0 \end{array}\right. \cup \left|\begin{array}{l} 0<x^2<1 \\ x^2+x-1>1 \end{array}\right. \iff$$

$$\left|\begin{array}{l} x<-1 \text{ или } x>1 \\ x^2+x-2<0 \\ x^2+x-1>0 \end{array}\right. \cup \left|\begin{array}{l} -1<x<1,\ x\neq 0 \\ x^2+x-2>0 \iff (x+2)(x-1)>0 \end{array}\right.$$

$x^2+x-1=0 \iff x_{1,2}=\frac{-1\pm\sqrt{5}}{2}$



Н. р.

отговор $x\in\left(-2;\ -\frac{1-\sqrt{5}}{2}\right)$.

Тъй като $\left|\begin{array}{l} x>0 \\ x\neq 1 \end{array}\right. \iff \left|\begin{array}{l} \frac{1}{x}>0 \\ \frac{1}{x}\neq 1 \end{array}\right.$

17) $\log_x(x+1)<\log_{\frac{1}{x}}(2-x) \iff \log_x(x+1)<\frac{\log_x(2-x)}{\log_x\frac{1}{x}}$

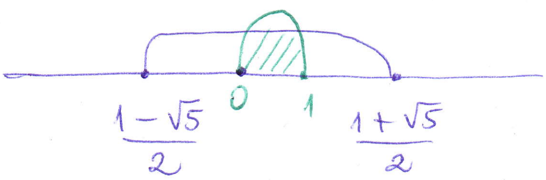
$\iff \log_x(x+1)<-\log_x(2-x) \iff$

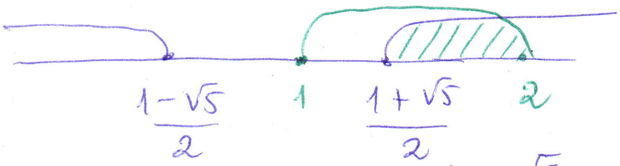
$\log_x(x+1)+\log_x(2-x)<\log_x 1 \iff$

$$\left|\begin{array}{l} x+1>0 \\ 2-x>0 \\ \log_x(x+1)(2-x)<\log_x 1 \end{array}\right. \iff \left|\begin{array}{l} -1<x<2 \\ \log_x(x+1)(2-x)<\log_x 1 \end{array}\right.$$

$$\iff \left|\begin{array}{l} 0<x<1 \\ (x+1)(2-x)>1 \end{array}\right. \cup \left|\begin{array}{l} 1<x<2 \\ (x+1)(2-x)<1 \end{array}\right. \iff$$

$$\left|\begin{array}{l} 0<x<1 \\ -x^2+x+2>1 \\ \Updownarrow \\ x^2-x-1<0 \end{array}\right. \cup \left|\begin{array}{l} 1<x<2 \\ x^2-x-1>0 \end{array}\right.$$





$x\in(0;1)\cup\left(\frac{1+\sqrt{5}}{2};\ 2\right)$

18) $\log_{4x}(x^2 - x - 2) > \log_{4x}(3 + 2x - x^2) \Leftrightarrow$

$$\left|\begin{array}{l} 4x > 1 \\ x^2 - x - 2 > 3 + 2x - x^2 \\ 3 + 2x - x^2 > 0 \end{array}\right. \cup \left|\begin{array}{l} 0 < 4x < 1 \\ x^2 - x - 2 < 3 + 2x - x^2 \\ x^2 - x - 2 > 0 \end{array}\right.$$

$$\Leftrightarrow \left|\begin{array}{l} x > \frac{1}{4} \\ 2x^2 - 3x - 5 > 0 \\ x^2 - 2x - 3 < 0 \end{array}\right. \cup \left|\begin{array}{l} 0 < x < \frac{1}{4} \\ 2x^2 - 3x - 5 < 0 \\ x^2 - x - 2 > 0 \Leftrightarrow (x-2)(x+1) > 0 \end{array}\right.$$

$\Updownarrow$ $\qquad\qquad$ $\Updownarrow$

н.р.

$$\left|\begin{array}{l} x > \frac{1}{4} \\ (x-3)(x+1) < 0 \\ 2\left(x - \frac{5}{2}\right)(x+1) > 0 \end{array}\right. \Leftrightarrow \underline{x \in \left(\frac{5}{2}; 3\right)}$$ – отговор на задачата

19) $\log_{\frac{3x-1}{x+2}}(2x^2 + x - 1) \geq \log_{\frac{3x-1}{x+2}}(11x - 6 - 3x^2) \Leftrightarrow$

$$\left|\begin{array}{l} \frac{3x-1}{x+2} > 1 \\ 2x^2 + x - 1 \geq 11x - 6 - 3x^2 \\ 11x - 6 - 3x^2 > 0 \end{array}\right. \cup \left|\begin{array}{l} 0 < \frac{3x-1}{x+2} < 1 \\ 2x^2 + x - 1 \leq 11x - 6 - 3x^2 \\ 2x^2 + x - 1 > 0 \end{array}\right.$$

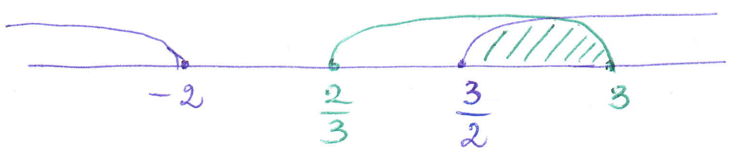
$$\left|\begin{array}{l} \frac{2x-3}{x+2} > 0 \\ (x-1)^2 \geq 0 \ \forall x \in \mathbb{R} \\ 3x^2 - 11x + 6 < 0 \end{array}\right. \cup \left|\begin{array}{l} \frac{3x-1}{x+2} > 0, \ \frac{2x-3}{x+2} < 0 \\ (x-1)^2 \leq 0 \Leftrightarrow x = 1 \\ 2x^2 + x - 1 > 0 \end{array}\right.$$

удовлетворява останалите условия в с-мата

$3(x-3)\left(x - \frac{2}{3}\right) < 0$

$3x^2 - 11x + 6 = 0$

$x_{1,2} = \frac{11 \pm 7}{6}$ $\nearrow x_1 = 3$, $\searrow x_2 = \frac{2}{3}$

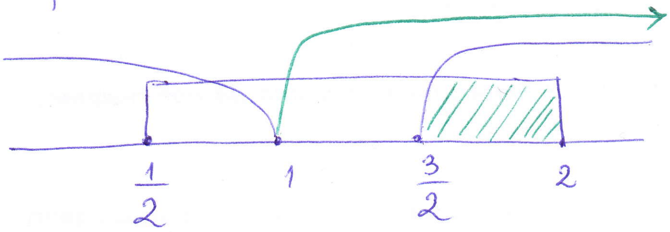


$x \in \left(\frac{3}{2}; 3\right) \cup \{1\}$

20) $\log_{\frac{x+1}{x-1}}\left(2x^2-5x+3\right) \le 0 \iff \log_{\frac{x+1}{x-1}}\left(2x^2-5x+3\right) \le \log_{\frac{x+1}{x-1}} 1$

$$\iff \left|\begin{array}{l} \frac{x+1}{x-1} > 1 \\ 2x^2-5x+3 \le 1 \\ 2x^2-5x+3 > 0 \end{array}\right. \cup \left|\begin{array}{l} 0 < \frac{x+1}{x-1} < 1 \\ 2x^2-5x+3 \ge 1 \end{array}\right. \iff$$

$$\left|\begin{array}{l} \frac{2}{x-1} > 0 \iff x > 1 \\ 2(x-2)\left(x-\frac{1}{2}\right) \le 0 \\ 2\left(x-\frac{3}{2}\right)(x-1) > 0 \end{array}\right. \cup \left|\begin{array}{l} \frac{x+1}{x-1} > 0, \quad \frac{2}{x-1} < 0 \\ 2(x-2)\left(x-\frac{1}{2}\right) \ge 0 \end{array}\right.$$



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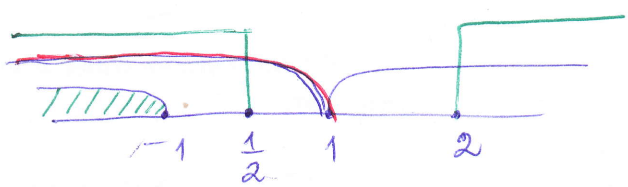


$x \in \left(\frac{3}{2}; 2\right]$ $\qquad$ $x \in (-\infty; -1)$

Отговор: $x \in (-\infty; -1) \cup \left(\frac{3}{2}; 2\right]$

21) С полагане $\log_2^2(x+1) - 3\log_2(x+1) \ge -2$

Полагане $\log_2(x+1) = t \Rightarrow t^2 - 3t + 2 \ge 0$

$(t-2)(t-1) \ge 0$

$\Rightarrow t \le 1$ или $t \ge 2 \iff$

$\log_2(x+1) \le 1$ или $\log_2(x+1) \ge 2 \iff$

$\log_2(x+1) \le \log_2 2$ или $\log_2(x+1) \ge \log_2 4 \iff$

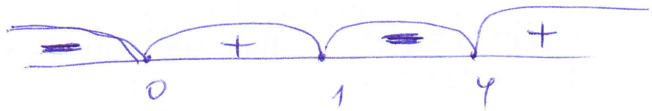
$$\left|\begin{array}{l} x+1 \le 2 \\ x+1 > 0 \end{array}\right. \cup \quad x+1 \ge 4 \iff$$

$$\left|\begin{array}{l} x \le 1 \\ x > -1 \end{array}\right. \cup \quad x \ge 3 \Rightarrow x \in (-1; 1] \cup [3; +\infty)$$

22) $\dfrac{\lg^2 x - 3\lg x - 1}{\lg x - 1} < 1$. Полагаме $\lg x = t \Rightarrow$

$$\frac{t^2 - 3t - 1}{t - 1} < 1 \Leftrightarrow \frac{t^2 - 3t - 1 - t + 1}{t - 1} < 0 \Leftrightarrow$$

$$\frac{t(t-4)}{t-1} < 0 \Leftrightarrow$$



$\Rightarrow \quad t < 0$ или $1 < t < 4 \Leftrightarrow$

$\lg x < 0$ или $1 < \lg x < 4 \Leftrightarrow$

$\lg x < \lg 1$ или $\lg 10 < \lg x < \lg 10^4 \Leftrightarrow$

$\left|\begin{array}{l} x < 1 \\ x > 0 \end{array}\right.$ или $10 < x < 10^4 \Rightarrow$

Отговор $x \in (0; 1) \cup (10; 10^4)$.

23) Свойства на функции

$$11^{10x-12} + \log_{11}(10x - 9) \le 11^{x+6} + \log_{11}(x+9)$$

Д.О. $\left|\begin{array}{l} 10x - 9 > 0 \Leftrightarrow x > \frac{9}{10} \\ x + 9 > 0 \Leftrightarrow x > -9 \end{array}\right. \Leftrightarrow x > \frac{9}{10}$

Изпълнено $10x - 12 \le x + 6 \Leftrightarrow 9x \le 18 \Leftrightarrow x \le 2$ и

$10x - 9 \le x + 9 \Leftrightarrow 9x \le 18 \Leftrightarrow x \le 2$.

Тогава при $x \in \left(\frac{9}{10}; 2\right]$ е в сила:

$\left|\begin{array}{l} 11^{10x-12} \le 11^{x+6} \\ \log_{11}(10x-9) \le \log_{11}(x+9) \end{array}\right.$ тъй като $y = 11^x$ и $y = \log_{11} x$ $\Rightarrow$ са растящи функции.

неравенството е изпълнено за $x \in \left(\frac{9}{10}; 2\right]$.

24) $\log_{x+2}\left(7x^2-x^3\right) + \log_{\frac{1}{x+2}}\left(x^2-3x\right) \geq \log_{\sqrt{x+2}}\sqrt{5-x}$

Тъй като $\left|\begin{array}{l} \frac{1}{x+2} > 0 \\ \frac{1}{x+2} \neq 1 \end{array}\right.$ и $\left|\begin{array}{l} \sqrt{x+2} > 0 \\ \sqrt{x+2} \neq 1 \end{array}\right. \Leftrightarrow \left|\begin{array}{l} x+2 > 0 \\ x+2 \neq 1 \end{array}\right.$)

то можем да преминем към еднаква основа и да получим следното еквивалентно уравнение:

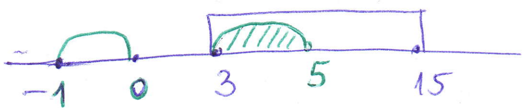
$$\log_{x+2}\left(7x^2-x^3\right) - \log_{x+2}\left(x^2-3x\right) \geq 2\log_{x+2}\sqrt{5-x} \Leftrightarrow$$

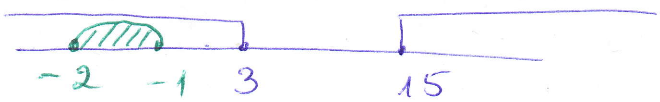
$$\left|\begin{array}{l} 7x^2-x^3>0 \Leftrightarrow x^2(x-7)<0 \Leftrightarrow x\neq 0,\ x<7 \\ x^2-3x>0 \Leftrightarrow x(x-3)>0 \Leftrightarrow x<0 \text{ или } x>3 \\ \log_{x+2}\dfrac{7x^2-x^3}{x^2-3x} \geq 2\log_{x+2}\sqrt{5-x} \end{array}\right. \Leftrightarrow$$

$$\left|\begin{array}{l} x\in(-\infty;0)\cup(3;7) \\ \log_{x+2}\dfrac{7x^2-x^3}{x^2-3x} \geq \log_{x+2}(5-x) \end{array}\right. \Leftrightarrow$$

$$\left|\begin{array}{l} x\in(-\infty;0)\cup(3;7) \\ x+2>1 \Leftrightarrow x>-1 \\ \dfrac{7x^2-x^3}{x^2-3x} \geq 5-x \\ 5-x>0 \Leftrightarrow x<5 \end{array}\right. \cup \left|\begin{array}{l} x\in(-\infty;0)\cup(3;7) \\ 0<x+2<1 \Leftrightarrow -2<x<-1 \\ \dfrac{7x^2-x^3}{x^2-3x} \leq 5-x \end{array}\right.$$

$$\left|\begin{array}{l} x\in(-1;0)\cup(3;5) \\ \dfrac{x(x-15)}{x(x-3)} \leq 0 \end{array}\right. \cup \left|\begin{array}{l} x\in(-2;-1) \\ \dfrac{x(x-15)}{x(x-3)} \geq 0 \end{array}\right.$$





$x\in(3;5) \quad \cup \quad x\in(-2;-1) \Rightarrow$

Отговор $x\in(-2;-1)\cup(3;5)$

25) $\log_2(4^x+4) < \log_2 2^x + \log_2(2^{x+1}-3) \Leftrightarrow$

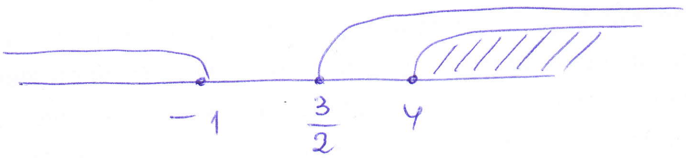
$$\begin{cases} \log_2(4^x+4) < \log_2(2^{x+1}-3)2^x \\ 2^{x+1}-3>0 \Leftrightarrow \quad 2^x > \frac{3}{2} \end{cases}$$

Нека положим $2^x = t > 0 \Rightarrow$

$$\begin{cases} \log_2(t^2+4) < \log_2 t(2t-3) \\ t > \frac{3}{2} \end{cases} \Leftrightarrow$$

$$\begin{cases} t^2+4 < t(2t-3) \\ t > \frac{3}{2} \end{cases} \Leftrightarrow \begin{cases} t^2+4 < 2t^2-3t \\ t > \frac{3}{2} \end{cases} \Leftrightarrow$$

$$\begin{cases} t^2-3t-4>0 \\ t > \frac{3}{2} \end{cases} \Leftrightarrow \begin{cases} (t-4)(t+1)>0 \\ t > \frac{3}{2} \end{cases}$$



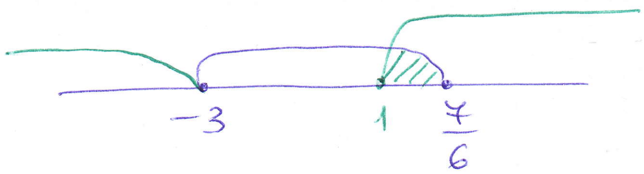
$\Rightarrow \quad t > 4 \Rightarrow$

$2^x > 4 \Rightarrow \underline{x > 2}$

26) $4^{\log_5 \frac{x-1}{x+3}} < \frac{1}{16} \Leftrightarrow 4^{\log_5 \frac{x-1}{x+3}} < 4^{-2} \Leftrightarrow$

$\log_5 \frac{x-1}{x+3} < -2 \Leftrightarrow \log_5 \frac{x-1}{x+3} < \log_5 \frac{1}{25} \Leftrightarrow$

$$\begin{cases} \frac{x-1}{x+3} < \frac{1}{25} \\ \frac{x-1}{x+3} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{6x-7}{x+3} < 0 \\ \frac{x-1}{x+3} > 0 \end{cases}$$



$\underline{x \in \left(1; \frac{7}{6}\right)}$

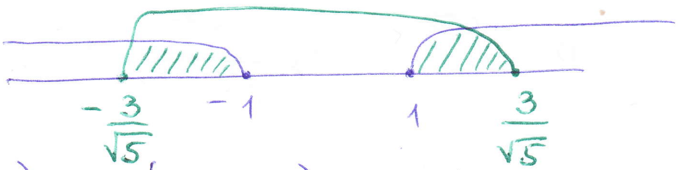
28) $\left(\frac{1}{2}\right)^{\log_3 \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right)} > 1$

$\left(\frac{1}{2}\right)^{\log_3 \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right)} > \left(\frac{1}{2}\right)^0 \Leftrightarrow \log_3 \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) < 0$

$\Leftrightarrow \log_3 \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) < \log_3 1 \Leftrightarrow$

$\left|\begin{array}{l} \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) < 1 \\ \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) > 0 \end{array}\right. \Leftrightarrow \left|\begin{array}{l} \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) < \log_{\frac{1}{5}} \frac{1}{5} \\ \log_{\frac{1}{5}}\left(x^2-\frac{7}{5}\right) > \log_{\frac{1}{5}} 1 \end{array}\right. \Leftrightarrow$

$\left|\begin{array}{l} x^2-\frac{7}{5} > \frac{1}{5} \\ x^2-\frac{7}{5} < 1 \end{array}\right. \Leftrightarrow \left|\begin{array}{l} x^2-1 > 0 \\ x^2-\frac{9}{5} < 0 \end{array}\right. \Leftrightarrow \left|\begin{array}{l} (x-1)(x+1) > 0 \\ \left(x-\frac{3}{\sqrt5}\right)\left(x+\frac{3}{\sqrt5}\right) < 0 \end{array}\right.$



$x \in \left(-\frac{3}{\sqrt5}; -1\right) \cup \left(1; \frac{3}{\sqrt5}\right).$

29) $5^{\log_5^2 x} + x^{\log_5 x} < 10$

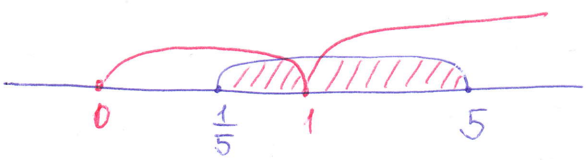
Д.О. $x > 0, \ x \neq 1 \Rightarrow \left(\underbrace{5^{\log_5 x}}_{x}\right)^{\log_5 x} + x^{\log_5 x} < 10$

$\Leftrightarrow 2x^{\log_5 x} < 10 \Leftrightarrow$

$x^{\log_5 x} < 5 \ \Big|. \log_5 \Leftrightarrow \log_5 x^{\log_5 x} < \log_5 5 \Leftrightarrow$

$(\log_5 x)^2 < 1 \Leftrightarrow -1 < \log_5 x < 1 \Leftrightarrow$

$\log_5 \frac{1}{5} < \log_5 x < \log_5 5 \Leftrightarrow \frac{1}{5} < x < 5$



$\Rightarrow x \in \left(\frac{1}{5}; 1\right) \cup (1; 5).$

Другият подход е за $x = 1$ да проверим дали удовлетворява даденото неравенство и в такъв случай да го причислим към решенията.

30) $\left(3^{x+2}+3^{-x}\right)^{3\lg x-\lg(2x^2+3x)}<1 \iff$

$$\left|\begin{array}{l} x>0 \\ 2x^2+3x>0 \iff x(2x+3)>0 \\ \left(3^{x+2}+3^{-x}\right)^{\lg\frac{x^3}{2x^2+3x}}<1 \end{array}\right. \quad \left.\begin{array}{l} \\ \\ \end{array}\right\} \iff x>0$$

при $x>0$ имаме $3^x>3^0=1 \Rightarrow 3^{x+2}=9.3^x>9 \Rightarrow$

$3^{x+2}+3^{-x}>1 \Rightarrow$

$$\left|\begin{array}{l} x>0 \\ \left(3^{x+2}+3^{-x}\right)^{\lg\frac{x^3}{2x^2+3x}}<\left(3^{x+2}+3^{-x}\right)^0 \end{array}\right. \iff$$

$$\left|\begin{array}{l} x>0 \\ \lg\dfrac{x^3}{2x^2+3x}<0 \end{array}\right. \iff \left|\begin{array}{l} x>0 \\ \lg\dfrac{x^3}{2x^2+3x}<\lg 1 \end{array}\right. \iff$$

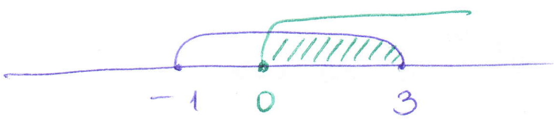
$$\left|\begin{array}{l} x>0 \\ \dfrac{x^3}{2x^2+3x}<1 \end{array}\right. \iff$$

можем да освободим от знаменател поради условието $2x^2+3x>0$

$x^3<2x^2+3x \iff x(x^2-2x-3)<0$,

което поради $x>0$ е еквивалентно на $x^2-2x-3<0$

$\iff (x-3)(x+1)<0$



$\Rightarrow$ $\underline{x\in(0;3)}$

$$8\sqrt{1-2^x+2^{2x-2}} > 2^{2x} - 2^{x+2} + 7$$

Нека положим $2^x = t > 0 \Rightarrow$

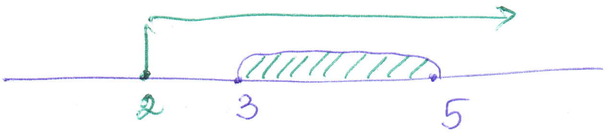
$$8\sqrt{1-t+\frac{t^2}{4}} > t^2-4t+7 \iff 8\sqrt{\frac{t^2-4t+4}{4}} > t^2-4t+7$$

$$\iff 4\sqrt{(t-2)^2} > t^2-4t+7 \iff 4|t-2| > t^2-4t+7$$

- при $t \geq 2$ имаме $|t-2| = t-2$ и неравенството приема вида

$4(t-2) > t^2-4t+7 \iff t^2-8t+15<0 \iff$

$(t-3)(t-5)<0 \iff 3<t<5$

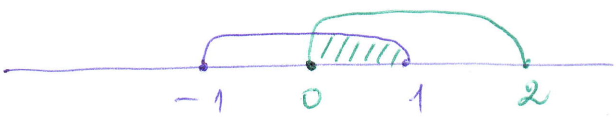


$\Rightarrow t \in (3;5)$ са решения

- при $t \in (0;2) \Rightarrow |t-2| = 2-t$ и неравенството приема вида

$4(2-t) > t^2-4t+7 \iff t^2-1<0 \iff (t-1)(t+1)<0$

$\iff -1<t<1$



$\Rightarrow t \in (0;1)$ са решения

Връщаме се в полагането

- от $t<1 \iff 2^x < 1 = 2^0 \iff x<0$

- от $3<t<5 \iff 3<2^x<5 \iff \log_2 3 < x < \log_2 5$

$\Rightarrow$ Отговор $x \in (-\infty; 0) \cup (\log_2 3; \log_2 5)$