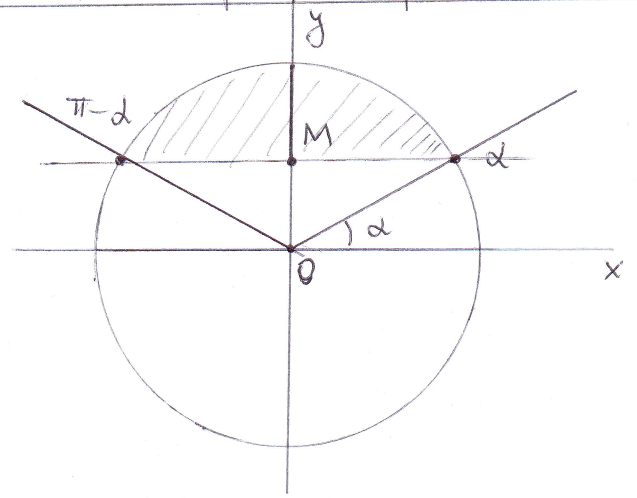
Тригонометрични неравенства

1. Основни тригонометрични неравенства



$$\sin x > \sin\alpha$$

т. $M(0;\ \sin\alpha)$

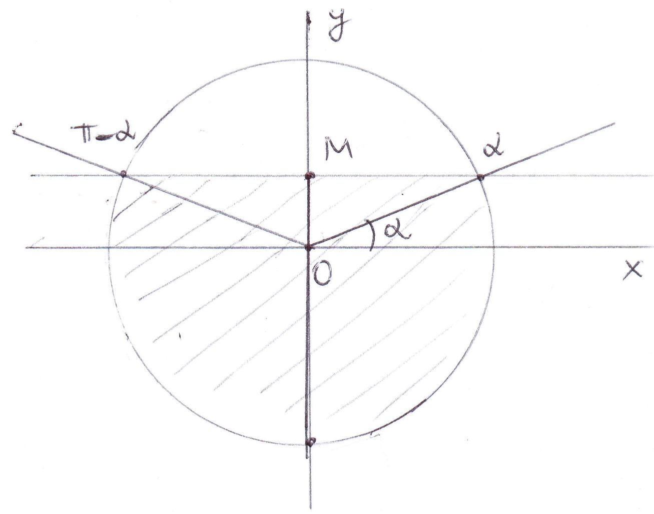
основни решения

$$\alpha < x < \pi - \alpha$$

$$\Downarrow$$

всички решения

$$\alpha + 2k\pi < x < \pi - \alpha + 2k\pi\ ,\ k \in \mathbb{Z}$$



$$\sin x < \sin\alpha$$

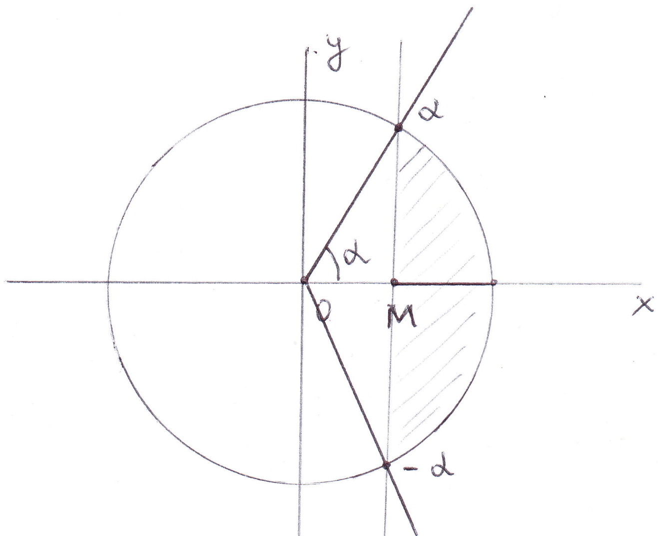
основни решения

$$\pi - \alpha < x < \alpha + 2\pi$$

$$\Downarrow$$

всички решения

$$\pi - \alpha + 2k\pi < x < \alpha + 2(k+1)\pi$$



$$\cos x > \cos\alpha$$

основни решения

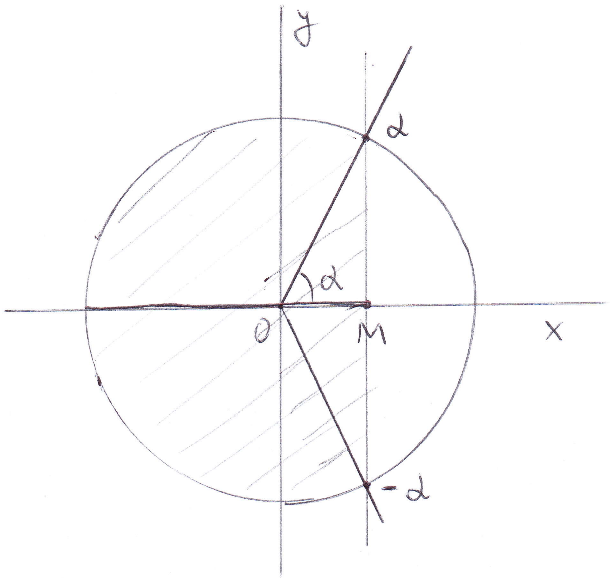
$$-\alpha < x < \alpha$$

$$\Downarrow$$

всички решения

$$-\alpha + 2k\pi < x < \alpha + 2k\pi$$

т. $M(\cos\alpha;\ 0)$



$\cos x < \cos\alpha$

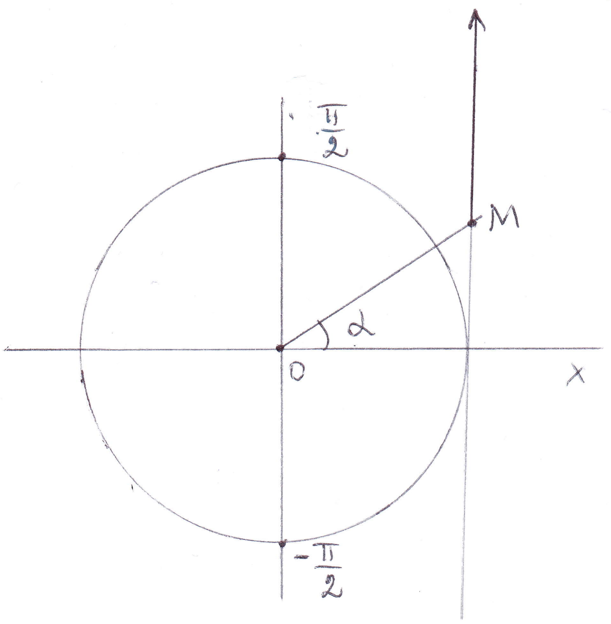
основни решения

$\alpha < x < 2\pi - \alpha$

$\Downarrow$

всички решения

$\alpha + 2k\pi < x < -\alpha + 2(k+1)\pi$



$\operatorname{tg} x > \operatorname{tg}\alpha$

т. $M(1;\ \operatorname{tg}\alpha)$

основни решения

$\alpha < x < \frac{\pi}{2}$

$\Downarrow$

всички решения

$\alpha + k\pi < x < \frac{\pi}{2} + k\pi$

$\operatorname{tg} x < \operatorname{tg}\alpha$

основни решения

$-\frac{\pi}{2} < x < \alpha$

$\Downarrow$

всички решения

$-\frac{\pi}{2} + k\pi < x < \alpha + k\pi$

2. Задачи

1) $\underline{\sin x \cos x \cos 2x < \frac{1}{8}} \iff 2\sin x \cos x \cos 2x < \frac{1}{4} \iff$

$\sin 2x \cos 2x < \frac{1}{4} \iff 2\sin 2x \cos 2x < \frac{1}{2} \iff$

$\sin 4x < \frac{1}{2} \iff \frac{5\pi}{6} + 2k\pi < 4x < \frac{\pi}{6} + 2(k+1)\pi \iff$

$\frac{5\pi}{24} + k\frac{\pi}{2} < x < \frac{\pi}{24} + (k+1)\frac{\pi}{2}$

2) $\underline{\cos x - \sin x > 1} \iff \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x > \frac{\sqrt{2}}{2} \iff$

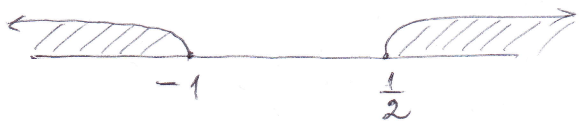
$\cos\left(x + \frac{\pi}{4}\right) > \frac{\sqrt{2}}{2} \iff -\frac{\pi}{4} + 2k\pi < x + \frac{\pi}{4} < \frac{\pi}{4} + 2k\pi$

$\iff -\frac{\pi}{2} + 2k\pi < x < 2k\pi$

3) $\underline{\cos 2x + \cos x > 0} \iff 2\cos^2 x + \cos x - 1 > 0$

Полагаме $\cos x = t \in [-1; 1] \Rightarrow 2t^2 + t - 1 > 0 \iff$

$2(t+1)\left(t - \frac{1}{2}\right) > 0$



$\Updownarrow$

$t < -1 \cup t > \frac{1}{2} \iff \underset{\text{н.р.}}{\cos x < -1} \cup \cos x > \frac{1}{2} \iff$

$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$

4) $\underline{\cos^3 x \cos 3x - \sin^3 x \sin 3x > \frac{5}{8}}, \quad x \in \left(0; \frac{\pi}{4}\right)$

$\cos^3 x\left(4\cos^3 x - 3\cos x\right) - \sin^3 x\left(3\sin x - 4\sin^3 x\right) > \frac{5}{8}$

$4\cos^6 x - 3\cos^4 x - 3\sin^4 x + 4\sin^6 x > \frac{5}{8}$

От $\sin^2 x + \cos^2 x = 1 \Rightarrow$

$$1 = (\sin^2 x + \cos^2 x)^3 = \sin^6 x + \cos^6 x + 3\sin^4 x \cos^2 x + 3\sin^2 x \cos^4 x \Rightarrow$$

$$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) \Rightarrow$$

$$\underline{\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x}$$

$$1 = (\sin^2 x + \cos^2 x)^2 = \sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x \Rightarrow$$

$$\underline{\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cos^2 x} \Rightarrow$$

неравенството е еквивалентно на

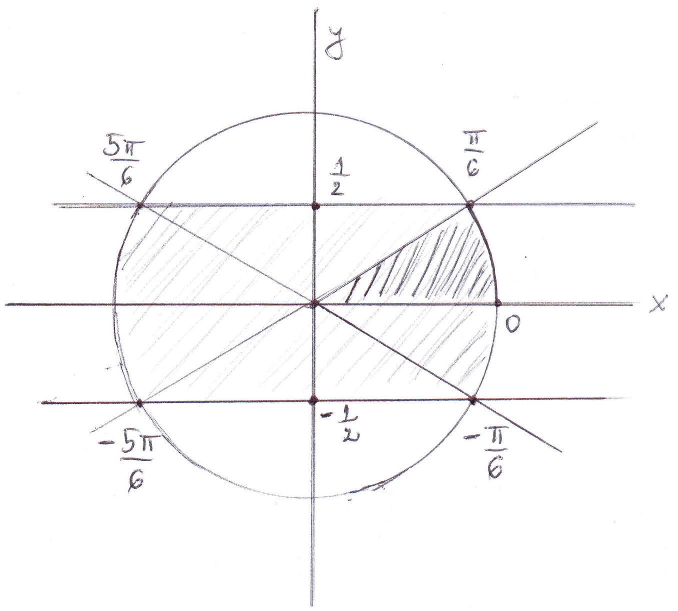
$$4(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) > \frac{5}{8} \Leftrightarrow$$

$$4(1 - 3\sin^2 x \cos^2 x) - 3(1 - 2\sin^2 x \cos^2 x) > \frac{5}{8} \Leftrightarrow$$

$$1 - 6\sin^2 x \cos^2 x > \frac{5}{8} \Leftrightarrow 4\sin^2 x \cos^2 x < \frac{1}{4} \Leftrightarrow$$

$$\sin^2 2x < \frac{1}{4} \Leftrightarrow -\frac{1}{2} < \sin 2x < \frac{1}{2} \text{ и } x \in \left(0; \frac{\pi}{4}\right)$$

(по условие)



$\Updownarrow$

$$0 < 2x < \frac{\pi}{6} \Leftrightarrow$$

$$\underline{0 < x < \frac{\pi}{12}.}$$