ALMOST COMPLEX CONNECTIONS ON ALMOST COMPLEX MANIFOLDS WITH NORDEN METRIC

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A four-parametric family of linear connections preserving the almost complex structure is defined on an almost complex manifold with Norden metric. Necessary and sufficient conditions for these connections to be natural are obtained. A two-parametric family of complex connections is studied on a conformal Kähler manifold with Norden metric. The curvature tensors of these connections are proved to coincide.

Introduction

Almost complex manifolds with Norden metric were first studied by A. P. Norden [9]. These manifolds are introduced in [6] as generalized B-manifolds. A classification of the considered manifolds with respect to the covariant derivative of the almost complex structure is obtained in [2] and two equivalent classifications are given in [3,5].

An important problem in the geometry of almost complex manifolds with Norden metric is the study of linear connections preserving the almost complex structure or preserving both, the structure and the Norden metric. The first ones are called *almost complex* connections, and the second ones are called *natural* connections. A special type of a natural connection is the canonical one. In [3] it is proved that on an almost complex manifold with Norden metric there exists a unique canonical connection. The canonical connection and its conformal group on a conformal Kähler manifold with Norden metric are studied in [5].

In [10] we have studied the Yano connection on a complex manifold

*The author is supported by The Fund for Scientific Research of the University of Plovdiv "Paisii Hilendarski", Project for young researchers MU-5, contract No. 494/08.
Mathematics Subject Classification: Primary 53C15, 53C50; Secondary 32Q60.

with Norden metric. In [11] we have proved that the curvature tensors of the canonical connection and the Yano connection coincide on a conformal Kähler manifold with Norden metric.

In the present paper we define a four-parametric family of almost complex connections on an almost complex manifold with Norden metric. We find necessary and sufficient conditions for these connections to be natural. By this way we obtain a two-parametric family of natural connections on an almost complex manifold with Norden metric. We study a two-parametric family of complex connections on a conformal Kähler manifold with Norden metric, obtain the form of the Kähler curvature tensor corresponding to each of these connections and prove that these tensors coincide.

1. Preliminaries

Let (M, J, g) be a $2n$-dimensional almost complex manifold with Norden metric, i.e. J is an almost complex structure and g is a metric on M such that

$$J^2X = -X, \qquad g(JX, JY) = -g(X, Y) \tag{1}$$

for all differentiable vector fields X, Y on M, i.e. $X, Y \in \mathfrak{X}(M)$.

The associated metric $\widetilde{g}$ of g, given by $\widetilde{g}(X, Y) = g(X, JY)$, is a Norden metric, too. Both metrics are necessarily neutral, i.e. of signature (n, n).

Further, X, Y, Z, W $(x, y, z, w$, respectively) will stand for arbitrary differentiable vector fields on M (vectors in T_pM, $p \in M$, respectively).

If ∇ is the Levi-Civita connection of the metric g, the tensor field F of type $(0, 3)$ on M is defined by $F(X, Y, Z) = g\left((\nabla_X J)Y, Z\right)$ and has the following symmetries

$$F(X, Y, Z) = F(X, Z, Y) = F(X, JY, JZ). \tag{2}$$

Let $\{e_i\}$ $(i = 1, 2, \ldots, 2n)$ be an arbitrary basis of T_pM at a point p of M. The components of the inverse matrix of g are denoted by g^{ij} with respect to the basis $\{e_i\}$. The Lie 1-forms θ and θ^* associated with F, and the Lie vector Ω, corresponding to θ, are defined by, respectively

$$\theta(z) = g^{ij}F(e_i, e_j, z), \qquad \theta^* = \theta \circ J, \qquad \theta(z) = g(z, \Omega). \tag{3}$$

The Nijenhuis tensor field N for J is given by $N(X, Y) = [JX, JY] - [X, Y] - J[JX, Y] - J[X, JY]$. The corresponding tensor of type (0,3) is given by $N(X, Y, Z) = g(N(X, Y), Z)$. In terms of ∇J this tensor is expressed in the following way

$$N(X, Y) = (\nabla_X J)JY - (\nabla_Y J)JX + (\nabla_{JX} J)Y - (\nabla_{JY} J)X. \tag{4}$$

It is known [8] that the almost complex structure is complex if and only if it is integrable, i.e. $N = 0$. The associated tensor $\widetilde{N}$ of N is defined by [2]

$$\widetilde{N}(X,Y) = (\nabla_X J)JY + (\nabla_Y J)JX + (\nabla_{JX} J)Y + (\nabla_{JY} J)X, \qquad (5)$$

and the corresponding tensor of type (0,3) is given by $\widetilde{N}(X,Y,Z) = g(\widetilde{N}(X,Y),Z)$.

A classification of the almost complex manifolds with Norden metric is introduced in [2], where eight classes of these manifolds are characterized according to the properties of F. The three basic classes $\mathcal{W}_i$ $(i = 1,2,3)$ and the class $\mathcal{W}_1 \oplus \mathcal{W}_2$ are given by

- the class $\mathcal{W}_1$:

$$\begin{aligned} F(X,Y,Z) = \tfrac{1}{2n} [&g(X,Y)\theta(Z) + g(X,JY)\theta(JZ) \\ &+g(X,Z)\theta(Y) + g(X,JZ)\theta(JY)]\,; \end{aligned} \qquad (6)$$

- the class $\mathcal{W}_2$ of *the special complex manifolds with Norden metric*:

$$F(X,Y,JZ) + F(Y,Z,JX) + F(Z,X,JY) = 0, \quad \theta = 0; \qquad (7)$$

- the class $\mathcal{W}_3$ of *the quasi-Kähler manifolds with Norden metric*:

$$F(X,Y,Z) + F(Y,Z,X) + F(Z,X,Y) = 0 \Leftrightarrow \widetilde{N} = 0; \qquad (8)$$

- the class $\mathcal{W}_1 \oplus \mathcal{W}_2$ of *the complex manifolds with Norden metric*:

$$F(X,Y,JZ) + F(Y,Z,JX) + F(Z,X,JY) = 0 \Leftrightarrow N = 0. \qquad (9)$$

The special class $\mathcal{W}_0$ of *the Kähler manifolds with Norden metric* is characterized by $F = 0$.

A $\mathcal{W}_1$-manifold with closed Lie 1-forms θ and θ^* is called *a conformal Kähler manifold with Norden metric*.

Let R be the curvature tensor of ∇, i.e. $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$ and $R(X,Y,Z,W) = g\,(R(X,Y)Z,W)$.

A tensor L of type (0,4) is said to be *curvature-like* if it has the properties of R, i.e. $L(X,Y,Z,W) = -L(Y,X,Z,W) = -L(X,Y,W,Z)$, $L(X,Y,Z,W) + L(Y,Z,X,W) + L(Z,X,Y,W) = 0$. Then, the Ricci tensor $\rho(L)$ and the scalar curvatures $\tau(L)$ and $\tau^*(L)$ of L are defined by:

$$\begin{gathered} \rho(L)(y,z) = g^{ij}L(e_i,y,z,e_j), \\ \tau(L) = g^{ij}\rho(L)(e_i,e_j), \quad \tau^*(L) = g^{ij}\rho(L)(e_i,Je_j). \end{gathered} \qquad (10)$$

A curvature-like tensor L is called *a Kähler tensor* if $L(X,Y,JZ,JW) = -L(X,Y,Z,W)$.

Let S be a tensor of type (0,2). We consider the following tensors [5]:

$$
\begin{aligned}
\psi_1(S)(X,Y,Z,W) &= g(Y,Z)S(X,W) - g(X,Z)S(Y,W) \\
&\quad + g(X,W)S(Y,Z) - g(Y,W)S(X,Z), \\
\psi_2(S)(X,Y,Z,W) &= g(Y,JZ)S(X,JW) - g(X,JZ)S(Y,JW) \\
&\quad + g(X,JW)S(Y,JZ) - g(Y,JW)S(X,JZ), \\
\pi_1 = \tfrac{1}{2}\psi_1(g), \qquad & \pi_2 = \tfrac{1}{2}\psi_2(g), \qquad \pi_3 = -\psi_1(\widetilde{g}) = \psi_2(\widetilde{g}).
\end{aligned}
\tag{11}
$$

The tensor $\psi_1(S)$ is curvature-like if S is symmetric, and the tensor $\psi_2(S)$ is curvature-like is S is symmetric and hybrid with respect to J, i.e. $S(X,JY) = S(Y,JX)$. The tensors $\pi_1 - \pi_2$ and π_3 are Kählerian.

2. Almost complex connections on almost complex manifolds with Norden metric

In this section we study almost complex connections and natural connections on almost complex manifolds with Norden metric. First, let us recall the following

Definition 2.1. [7] A linear connection ∇' on an almost complex manifold (M,J) is said to be *almost complex* if $\nabla' J = 0$.

Theorem 2.1. *On an almost complex manifold with Norden metric there exists a 4-parametric family of almost complex connections ∇' with torsion tensor T defined by, respectively:*

$$
\begin{aligned}
& g\big(\nabla'_X Y - \nabla_X Y, Z\big) = \tfrac{1}{2}F(X,JY,Z) + t_1\big\{F(Y,X,Z) \\
& +F(JY,JX,Z)\big\} + t_2\big\{F(Y,JX,Z) - F(JY,X,Z)\big\} \\
& +t_3\big\{F(Z,X,Y) + F(JZ,JX,Y)\big\} \\
& +t_4\big\{F(Z,JX,Y) - F(JZ,X,Y)\big\},
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
& T(X,Y,Z) = t_1\big\{F(Y,X,Z) - F(X,Y,Z) + F(JY,JX,Z) \\
& -F(JX,JY,Z)\big\} + \big(\tfrac{1}{2} - t_2\big)\big\{F(X,JY,Z) - F(Y,JX,Z)\big\} \\
& +t_2\big\{F(JX,Y,Z) - F(JY,X,Z)\big\} + 2t_3F(JZ,JX,Y) \\
& +2t_4F(Z,JX,Y),
\end{aligned}
\tag{13}
$$

where $t_i \in \mathbb{R}, \quad i = 1,2,3,4.$

Proof. *By (12), (2) and direct computation, we prove that $\nabla' J = 0$, i.e. the connections ∇' are almost complex.* □

By (8) and (9) we obtain the form of the almost complex connections ∇' on the manifolds belonging to the classes $\mathcal{W}_1 \oplus \mathcal{W}_2$ and $\mathcal{W}_3$ as follows, respectively

Corollary 2.1. *On a complex manifold with Norden metric there exists a 2-parametric family of complex connections ∇' defined by*

$$\begin{aligned}\nabla'_X Y = \nabla_X Y + \tfrac{1}{2}(\nabla_X J)JY + p\big\{(\nabla_Y J)X + (\nabla_{JY} J)JX\big\}\\ +q\big\{(\nabla_Y J)JX - (\nabla_{JY} J)X\big\},\end{aligned} \tag{14}$$

where $p = t_1 + t_3$, $q = t_2 + t_4$.

Corollary 2.2. *On a quasi-Kähler manifold with Norden metric there exists a 2-parametric family of almost complex connections ∇' defined by*

$$\begin{aligned}\nabla'_X Y = \nabla_X Y + \tfrac{1}{2}(\nabla_X J)JY + s\big\{(\nabla_Y J)X + (\nabla_{JY} J)JX\big\}\\ +t\big\{(\nabla_Y J)JX - (\nabla_{JY} J)X\big\},\end{aligned} \tag{15}$$

where $s = t_1 - t_3$, $t = t_2 - t_4$.

Definition 2.2. [3] A linear connection ∇' on an almost complex manifold with Norden metric (M, J, g) is said to be *natural* if

$$\nabla' J = \nabla' g = 0 \quad (\Leftrightarrow \nabla' g = \nabla' \widetilde{g} = 0). \tag{16}$$

Lemma 2.1. *Let (M, J, g) be an almost complex manifold with Norden metric and let ∇' be an arbitrary almost complex connection defined by (12). Then*

$$\begin{aligned}\big(\nabla'_X g\big)(Y,Z) = (t_2 + t_4)\widetilde{N}(Y,Z,X) - (t_1 + t_3)\widetilde{N}(Y,Z,JX),\\ \big(\nabla'_X \widetilde{g}\big)(Y,Z) = -(t_1 + t_3)\widetilde{N}(Y,Z,X) - (t_2 + t_4)\widetilde{N}(Y,Z,JX).\end{aligned} \tag{17}$$

Then, by help of Theorem 2.1 and Lemma 2.1 we prove

Theorem 2.2. *An almost complex connection ∇' defined by (12) is natural on an almost complex manifold with Norden metric if and only if $t_1 = -t_3$ and $t_2 = -t_4$, i.e.*

$$\begin{aligned}g\big(\nabla'_X Y - \nabla_X Y, Z\big) = \tfrac{1}{2}F(X, JY, Z)\\ +t_1 N(Y,Z,JX) - t_2 N(Y,Z,X).\end{aligned} \tag{18}$$

The equation (18) defines a 2-parametric family of natural connections on an almost complex manifold with Norden metric and non-integrable almost complex structure. In particular, by (8) and (17) for the manifolds in the class $\mathcal{W}_3$ we obtain

Corollary 2.3. *Let (M, J, g) be a quasi-Kähler manifold with Norden metric. Then, the connection ∇' defined by (15) is natural for all $s, t \in \mathbb{R}$.*

If (M, J, g) is a complex manifold with Norden metric, then from (9) and (18) it follows that there exists a unique natural connection ∇' in the family (14) which has the form

$$\nabla'_X Y = \nabla_X Y + \tfrac{1}{2}(\nabla_X J)JY. \qquad (19)$$

Definition 2.3. [3] A natural connection ∇' with torsion tensor T on an almost complex manifold with Norden metric is said to be *canonical* if

$$T(X, Y, Z) + T(Y, Z, X) - T(JX, Y, JZ) - T(Y, JZ, JX) = 0. \qquad (20)$$

Then, by applying the last condition to the torsion tensors of the natural connections (18), we obtain

Proposition 2.1. *Let (M, J, g) be an almost complex manifold with Norden metric. A natural connection ∇' defined by (18) is canonical if and only if $t_1 = 0$, $t_2 = \frac{1}{8}$. In this case (18) takes the form*

$$2g\big(\nabla'_X Y - \nabla_X Y, Z\big) = F(X, JY, Z) - \tfrac{1}{4}N(Y, Z, X).$$

Let us remark that G. Ganchev and V. Mihova [3] have proven that on an almost complex manifold with Norden metric there exists a unique canonical connection. The canonical connection of a complex manifold with Norden metric has the form (19).

Next, we study the properties of the torsion tensors (13) of the almost complex connections ∇'.

The torsion tensor T of an arbitrary linear connection is said to be *totally antisymmetric* if $T(X, Y, Z) = g(T(X, Y), Z)$ is a 3-form. The last condition is equivalent to

$$T(X, Y, Z) = -T(X, Z, Y). \qquad (21)$$

Then, having in mind (13) we obtain that the torsion tensors of the almost complex connections ∇' defined by (12) satisfy the condition (21) if and only if $t_1 = t_2 = t_3 = 0$, $t_4 = \frac{1}{4}$. Hence, we prove the following

Theorem 2.3. *Let (M, J, g) be an almost complex manifold with Norden metric and non-integrable almost complex structure. Then, on M there exists a unique almost complex connection ∇' in the family (12) whose torsion tensor is a 3-form. This connection is defined by*

$$g\big(\nabla'_X Y - \nabla_X Y, Z\big) = \tfrac{1}{4}\big\{2F(X, JY, Z) + F(Z, JX, Y) - F(JZ, X, Y)\big\}. \qquad (22)$$

By Corollary 2.2 and the last theorem we obtain

Corollary 2.4. *On a quasi-Kähler manifold with Norden metric there exists a unique natural connection ∇' in the family (15) whose torsion tensor is a 3-form. This connection is given by*

$$\nabla'_X Y = \nabla_X Y + \tfrac{1}{4}\big\{2(\nabla_X J)JY - (\nabla_Y J)JX + (\nabla_{JY} J)X\big\}. \qquad (23)$$

Let us remark that the connection (23) can be considered as an analogue of the Bismut connection [1], [4] in the geometry of the almost complex manifolds with Norden metric.

Let us consider symmetric almost complex connections in the family (12). By (13) and (4) we obtain

$$T(X,Y) - T(JX,JY) = \tfrac{1}{2}N(X,Y). \qquad (24)$$

From (24), (13) and (4) it follows that $T = 0$ if and only if $N = 0$ and $t_1 = t_3 = t_4 = 0$, $t_2 = \frac{1}{4}$. Then, it is valid the following

Theorem 2.4. *Let (M,J,g) be a complex manifold with Norden metric. Then, on M there exists a unique complex symmetric connection ∇' belonging to the family (14) which is given by*

$$\nabla'_X Y = \nabla_X Y + \tfrac{1}{4}\big\{(\nabla_X J)JY + 2(\nabla_Y J)JX - (\nabla_{JX} J)Y\big\}. \qquad (25)$$

The connection (25) is known as the Yano connection [12,13].

We give a summery of the obtained results for the 4-parametric family of almost complex connections ∇' in the following

	Class manifolds		
Connection type	$\mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$	$\mathcal{W}_1 \oplus \mathcal{W}_2$	$\mathcal{W}_3$
almost complex	$t_1, t_2, t_3, t_4 \in \mathbb{R}$	$p, q \in \mathbb{R}$	$s, t \in \mathbb{R}$
natural	$t_1 = -t_3$, $t_2 = -t_4$	$p = q = 0$	$s, t \in \mathbb{R}$
canonical	$t_1 = t_3 = 0$, $t_2 = -t_4 = \frac{1}{8}$	$p = q = 0$	$s = 0$, $t = \frac{1}{4}$
T is a 3-form	$t_1 = t_2 = t_3 = 0$, $t_4 = \frac{1}{4}$	$\nexists$	$s = 0$, $t = -\frac{1}{4}$
symmetric	$\nexists$	$p = 0$, $q = \frac{1}{4}$	$\nexists$

Table 1

3. Complex connections on conformal Kähler manifolds with Norden metric

Let (M,J,g) be a $\mathcal{W}_1$-manifold with Norden metric and consider the 2-parametric family of complex connections ∇' defined by (14). By (6) we obtain the form of ∇' on a $\mathcal{W}_1$-manifold as follows

$$\begin{aligned}\nabla'_X Y = \nabla_X Y + \tfrac{1}{4n}\big\{g(X,JY)\Omega - g(X,Y)J\Omega + \theta(JY)X\\ -\theta(Y)JX\big\} + \tfrac{p}{n}\big\{\theta(X)Y + \theta(JX)JY\big\} + \tfrac{q}{n}\big\{\theta(JX)Y - \theta(X)JY\big\}.\end{aligned} \tag{26}$$

Then, by (26) and straightforward computation we prove

Theorem 3.1. *Let (M,J,g) be a conformal Kähler manifold with Norden metric and ∇' be an arbitrary complex connection in the family (14). Then, the Kähler curvature tensor R' of ∇' has the form*

$$R' = R - \frac{1}{4n}\big\{\psi_1 + \psi_2\big\}(S) - \frac{1}{8n^2}\psi_1(P) - \frac{\theta(\Omega)}{16n^2}\big\{3\pi_1 + \pi_2\big\} + \frac{\theta(J\Omega)}{16n^2}\pi_3,$$

where S and P are defined by, respectively:

$$\begin{aligned}S(X,Y) = \big(\nabla_X\theta\big)JY + \tfrac{1}{4n}\big\{\theta(X)\theta(Y) - \theta(JX)\theta(JY)\big\},\\ P(X,Y) = \theta(X)\theta(Y) + \theta(JX)\theta(JY).\end{aligned} \tag{27}$$

By (26) we prove the following

Lemma 3.1. *Let (M,J,g) be a $\mathcal{W}_1$-manifold and ∇' be an arbitrary complex connection in the family (14). Then, the covariant derivatives of g and $\widetilde{g}$ are given by*

$$\begin{aligned}\big(\nabla'_X g\big)(Y,Z) = -\tfrac{2}{n}\big\{[p\theta(X) + q\theta(JX)]g(Y,Z)\\ +[p\theta(JX) - q\theta(X)]g(Y,JZ)\big\},\\ \big(\nabla'_X \widetilde{g}\big)(Y,Z) = \tfrac{2}{n}\big\{[p\theta(JX) - q\theta(X)]g(Y,Z)\\ -[p\theta(X) + q\theta(JX)]g(Y,JZ)\big\}.\end{aligned} \tag{28}$$

It is well-known [7] that the curvature tensor R' and the torsion tensor T of an arbitrary linear connection ∇' satisfy the second Bianchi identity, i.e.

$$\underset{X,Y,Z}{\mathfrak{S}}\big\{\big(\nabla'_X R'\big)(Y,Z,W) + R'\big(T(X,Y),Z,W\big)\big\} = 0, \tag{29}$$

where $\mathfrak{S}$ is the cyclic sum over X,Y,Z.

From (26) it follows that the torsion tensor of an arbitrary connection ∇' in the family (14) has the following form on a $\mathcal{W}_1$-manifold

$$\begin{aligned}T(X,Y) = \tfrac{1-4q}{4n}\big\{\theta(X)JY - \theta(Y)JX - \theta(JX)Y + \theta(JY)X\big\}\\ + \tfrac{p}{n}\big\{\theta(X)Y - \theta(Y)X + \theta(JX)JY - \theta(JY)JX\big\}\end{aligned} \tag{30}$$

Let us denote $\tau' = \tau(R')$ and $\tau'^{*} = \tau^{*}(R')$. We establish the following

Theorem 3.2. *Let (M,J,g) be a conformal Kähler manifold with Norden metric, and τ' and τ'^{*} be the scalar curvatures of the Kähler curvature* *tensor R' corresponding to the complex connection ∇' defined by (14). Then, the function $\tau' + i\tau'^*$ is holomorphic on M and the Lie 1-forms θ and θ^* are defined in a unique way by τ' and τ'^* as follows*:

$$\theta = 2n\,d\big(\arctan\frac{\tau'}{\tau'^*}\big), \qquad \theta^* = -2n\,d\big(\ln\sqrt{\tau'^2+\tau'^{*2}}\big). \tag{31}$$

Proof. *By (29) and (30) we obtain*

$$\begin{aligned}
&\big(\nabla'_X R'\big)(Y,Z,W) + \big(\nabla'_Y R'\big)(Z,X,W) + \big(\nabla'_Z R'\big)(X,Y,W)\\
&\quad= \tfrac{4q-1}{2n}\big\{\theta(X)R'(JY,Z,W) - \theta(JX)R'(Y,Z,W)\\
&\qquad -\theta(Y)R'(JX,Z,W) + \theta(JY)R'(X,Z,W)\\
&\qquad +\theta(Z)R'(JX,Y,W) - \theta(JZ)R'(X,Y,W)\big\}\\
&-\tfrac{2p}{n}\big\{\theta(X)R'(Y,Z,W) + \theta(JX)R'(JY,Z,W) - \theta(Y)R'(X,Z,W)\\
&-\theta(JY)R'(JX,Z,W) + \theta(Z)R'(X,Y,W) + \theta(JZ)R'(JX,Y,W)\big\}.
\end{aligned} \tag{32}$$

Then, having in mind the equalities (28) and their analogous equalities for g^{ij}, we find the total traces of the both sides of (32) and get

$$d\tau' = \tfrac{1}{2n}\big\{\tau'^*\theta - \tau'\theta^*\big\}, \qquad d\tau'^* = -\tfrac{1}{2n}\big\{\tau'\theta + \tau'^*\theta^*\big\}. \tag{33}$$

From (33) it follows immediately that $d\tau'^* \circ J = -d\tau'$, i.e. the function $\tau' + i\tau'^*$ is holomorphic on M and the equalities (31) hold. □

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