LIE GROUPS AS FOUR-DIMENSIONAL CONFORMAL KÄHLER MANIFOLDS WITH NORDEN METRIC

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An example of a four-dimensional conformal Kähler manifold with Norden metric is constructed on a Lie group. The form of the curvature tensor is obtained and the isotropic-Kähler properties of the manifold are studied.

Introduction

Almost complex manifolds with Norden metric are originally introduced in [7] as generalized B-manifolds. These manifolds are classified into eight classes in [3], and equivalent characteristic conditions for each of the classes are obtained in [4]. Examples of the basic classes of the integrable almost complex manifolds with Norden metric are given in [1]. An example of the only basic class of the considered manifolds with a non-integrable almost complex structure is introduced in [6].

In this paper we present an example of a four-dimensional conformal Kähler manifold with Norden metric which is obtained by constructing a four-parametric family of Lie algebras. We obtain the form of the curvature tensor and we study the conditions the given manifold to be isotropic Kählerian.

1. Almost complex manifolds with Norden metric

Let (M, J, g) be a $2n$-dimensional almost complex manifold with Norden metric, i.e. J is an almost complex structure and g is a metric on M such that

$$J^2X = -X, \qquad g(JX, JY) = -g(X, Y) \tag{1}$$

for all differentiable vector fields X, Y on M, i.e. $X, Y \in \mathfrak{X}(M)$.

The associated metric $\widetilde{g}$ of g, given by $\widetilde{g}(X,Y) = g(X,JY)$, is a Norden metric, too. Both metrics are necessarily neutral, i.e. of signature (n,n).

Further, X,Y,Z,W $(x,y,z,w$, respectively) will stand for arbitrary differentiable vector fields on M (vectors in T_pM, $p \in M$, respectively).

If ∇ is the Levi-Civita connection of the metric g, the tensor field F of type $(0,3)$ on M is defined by $F(X,Y,Z) = g\left((\nabla_X J)Y, Z\right)$ and has the following symmetries

$$F(X,Y,Z) = F(X,Z,Y) = F(X,JY,JZ). \tag{2}$$

Let $\{e_i\}$ $(i = 1,2,\dots,2n)$ be an arbitrary basis of T_pM at a point p of M. The components of the inverse matrix of g are denoted by g^{ij} with respect to the basis $\{e_i\}$. The Lie forms θ and θ^* associated with F, and the Lie vector Ω, corresponding to θ, are defined by, respectively

$$\theta(z) = g^{ij}F(e_i,e_j,z), \qquad \theta^* = \theta \circ J, \qquad \theta(z) = g(z,\Omega). \tag{3}$$

The Nijenhuis tensor field N is given as $N(X,Y) = [JX,JY] - [X,Y] - J[JX,Y] - J[X,JY]$. It is known [8] that the almost complex structure J is complex, if and only if $N = 0$.

A classification of the almost complex manifolds with Norden metric is introduced in [3], where eight classes of these manifolds are characterized according to the properties of F. The three basic classes and the class $\mathcal{W}_1 \oplus \mathcal{W}_2$ of the complex manifolds with Norden metric are given by:

$$\begin{aligned}
&\mathcal{W}_1 : F(X,Y,Z) = \tfrac{1}{2n}\left[g(X,Y)\theta(Z) + g(X,Z)\theta(Y)\right.\\
&\qquad\qquad\qquad\left. + g(X,JY)\theta(JZ) + g(X,JZ)\theta(JY)\right];\\
&\mathcal{W}_2 : F(X,Y,JZ) + F(Y,Z,JX) + F(Z,X,JY) = 0, \quad \theta = 0;\\
&\mathcal{W}_3 : F(X,Y,Z) + F(Y,Z,X) + F(Z,X,Y) = 0;\\
&\mathcal{W}_1 \oplus \mathcal{W}_2 : F(X,Y,JZ) + F(Y,Z,JX) + F(Z,X,JY) = 0.
\end{aligned} \tag{4}$$

The class $\mathcal{W}_0$ of the Kähler manifolds with Norden metric is given by $F = 0$.

Let R be the curvature tensor of ∇, i.e. $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$ and $R(X,Y,Z,W) = g\left(R(X,Y)Z,W\right)$.

The Ricci tensor ρ and the scalar curvatures τ and $\overset{*}{\tau}$ of R are given by:

$$\rho(y,z) = g^{ij}R(e_i,y,z,e_j), \quad \tau = g^{ij}\rho(e_i,e_j), \quad \overset{*}{\tau} = g^{ij}\rho(e_i,Je_j). \tag{5}$$

It is well known that the Weyl tensor W on a $2n$-dimensional pseudo-Riemannian manifold $(2n \geq 4)$ is determined by

$$W = R - \frac{1}{2n-2}\left\{\psi_1(\rho) - \frac{\tau}{2n-1}\pi_1\right\}, \tag{6}$$

where

$$\begin{aligned} \psi_1\big(\rho\big)(X,Y,Z,W) &= g(Y,Z)\rho(X,W) - g(X,Z)\rho(Y,W) \\ &\quad + g(X,W)\rho(Y,Z) - g(Y,W)\rho(X,Z), \\ \pi_1(X,Y,Z,W) &= g(Y,Z)g(X,W) - g(X,Z)g(Y,W). \end{aligned} \tag{7}$$

The Weyl tensor vanishes if and only if the manifold is conformally flat.

Let $\alpha = \{x, y\}$ be a non-degenerate two-plane spanned by the vectors $x, y \in T_pM$, $p \in M$. Then, the sectional curvature of α is given by:

$$\nu(\alpha; p) = \frac{R(x,y,y,x)}{\pi_1(x,y,y,x)}. \tag{8}$$

We consider the following basic sectional curvatures in T_pM with respect to the structures J and g: *holomorphic sectional curvatures* if $J\alpha = \alpha$ and *totally real sectional curvatures* if $J\alpha \perp \alpha$ with respect to g.

The square norm $\|\nabla J\|^2$ of ∇J is introduced in [5] by

$$\|\nabla J\|^2 = g^{ij} g^{kl} g\left((\nabla_{e_i} J)e_k, (\nabla_{e_j} J)e_l\right). \tag{9}$$

Then, the definition of F, (2) and (9) imply

$$\|\nabla J\|^2 = g^{ij} g^{kl} g^{pq} F_{ikp} F_{jlq}, \quad F_{ikp} = F(e_i, e_k, e_p). \tag{10}$$

Definition 1.1. [6] *An almost complex manifold with Norden metric, satisfying the condition* $\|\nabla J\|^2 = 0$, *is said to be* isotropic Kählerian.

It is known [9] that the curvature tensor R on any almost complex manifold with Norden metric satisfies the identity

$$(\nabla_X F)(Y,Z,JW) - (\nabla_Y F)(X,Z,JW) = R(X,Y,Z,W) + R(X,Y,JZ,JW). \tag{11}$$

Further, by (2) and (3) we obtain the following properties:

$$\begin{aligned} &\big(\nabla_X F\big)(Y,Z,W) = \big(\nabla_X F\big)(Y,W,Z); \\ &(\nabla_X F)(Y,JZ,W) = -(\nabla_X F)(Y,Z,JW) - g\left((\nabla_X J)Z, (\nabla_Y J)W\right) \\ &\qquad\qquad - g\left((\nabla_X J)W, (\nabla_Y J)Z\right); \\ &\big(\nabla_X \theta^*\big)Y = \big(\nabla_X \theta\big)JY + F(X,Y,\Omega);\ \theta(\Omega) = g^{ik} g^{jl} g\big((\nabla_{e_i} J)e_k, (\nabla_{e_j} J)e_l\big). \end{aligned} \tag{12}$$

Let us denote $\overset{**}{\tau} = g^{il} g^{jk} R(e_i, e_j, Je_k, Je_l)$. If R is a Kähler tensor, i.e. if $R(X,Y,JZ,JW) = -R(X,Y,Z,W)$, we have $\overset{**}{\tau} = -\tau$.

Let (M, J, g) be in $\mathcal{W}_1 \oplus \mathcal{W}_2$. Then, by (4) and (9) we get

$$2g^{il} g^{jk} g\big((\nabla_{e_i} J)e_k, (\nabla_{e_j} J)e_l\big) = \|\nabla J\|^2. \tag{13}$$

Theorem 1.1. *On a complex manifold with Norden metric it is valid*

$$\tau + \overset{**}{\tau} + \theta(\Omega) - 2\mathrm{div}(J\Omega) = \frac{1}{2}\left\|\nabla J\right\|^2, \tag{14}$$

where $\mathrm{div}(J\Omega) = \nabla_i J_k^i \Omega^k$.

Proof. By the properties (12), from (11) we obtain

$$\begin{aligned}&\big(\nabla_X F\big)(Y,Z,JW) + \big(\nabla_Y F\big)(X,W,JZ) + g\big((\nabla_X J)Z,(\nabla_Y J)W\big)\\ &+g\big((\nabla_X J)W,(\nabla_Y J)Z\big) = R(X,Y,Z,W) + R(X,Y,JZ,JW).\end{aligned} \tag{15}$$

Then, taking into account (12), (13) and $\nabla g = 0$, the total trace of (15) implies (14). ■

It has been proved [10] that on a $\mathcal{W}_1$-manifold with Norden metric it is valid $\left\|\nabla J\right\|^2 = \frac{2}{n}\theta(\Omega)$. Then, Theorem 1.1 induces

Corollary 1.1. *On a $\mathcal{W}_1$-manifold with Norden metric we have*

$$\tau + \overset{**}{\tau} - 2\mathrm{div}(J\Omega) = -\frac{n-1}{2}\left\|\nabla J\right\|^2. \tag{16}$$

The equality (16) and Definition 1.1 immediately imply

Corollary 1.2. *A $\mathcal{W}_1$-manifold with Norden metric is isotropic Kählerian if and only if* $\tau + \overset{**}{\tau} = 2\mathrm{div}(J\Omega)$.

Further, let us consider the class $\mathcal{W}_2$. By (4) and (14) it follows

Corollary 1.3. *On a $\mathcal{W}_2$-manifold with Norden metric it is valid*

$$2\big(\tau + \overset{**}{\tau}\big) = \left\|\nabla J\right\|^2. \tag{17}$$

Then, Corollary 1.3 and Definition 1.1 give rise to

Corollary 1.4. *A $\mathcal{W}_2$-manifold with Norden metric is isotropic Kählerian if its curvature tensor R is Kählerian.*

2. A Lie group as a four-dimensional conformal Kähler manifold with Norden metric

Let $\mathfrak{g}$ be a real four-dimensional Lie algebra corresponding to a real connected Lie group G. If $\{X_1, X_2, X_3, X_4\}$ is a global basis of left invariant vector fields on G and $[X_i, X_j] = C_{ij}^k X_k$, then the Jacobi identity is valid:

$$C_{ij}^k C_{ks}^l + C_{js}^k C_{ki}^l + C_{si}^k C_{kj}^l = 0. \tag{18}$$

We define an almost complex structure on G by the conditions:

$$JX_1 = X_3, \quad JX_2 = X_4, \quad JX_3 = -X_1, \quad JX_4 = -X_2. \tag{19}$$

Let us consider the left-invariant metric given by

$$\begin{gathered} g(X_1,X_1) = g(X_2,X_2) = -g(X_3,X_3) = -g(X_4,X_4) = 1, \\ g(X_i,X_j) = 0 \text{ for } i \neq j. \end{gathered} \tag{20}$$

The introduced metric is Norden because of (19). Hence the induced 4-dimensional manifold (G, J, g) is an almost complex manifold with Norden metric.

It is known [2] that an almost complex structure J on a Lie group G is said to be *abelian* if

$$[JX, JY] = [X, Y] \quad \text{for all } X, Y \in \mathfrak{g}. \tag{21}$$

From (21) we derive that the Nijenhuis tensor vanishes on $\mathfrak{g}$, i.e. J is a complex structure. Thus, (G, J, g) is a complex manifold with Norden metric.

The well-known equality

$$\begin{aligned} 2g(\nabla_X Y, Z) = &Xg(Y,Z) + Yg(X,Z) - Zg(X,Y) \\ &+g([X,Y],Z) + g([Z,X],Y) + g([Z,Y],X) \end{aligned} \tag{22}$$

implies

$$\begin{aligned} &2F(X_i,X_j,X_k) = g\left([X_i,JX_j] - J[X_i,X_j],X_k\right) \\ &+g\left([X_k,JX_i] - [JX_k,X_i],X_j\right) + g\left(J[X_k,X_j] - [JX_k,X_j],X_i\right). \end{aligned} \tag{23}$$

Let (G, J, g) be a $\mathcal{W}_1$-manifold. Then, by (3), (4), (21) and (23) we get

Lemma 2.1. *If (G, J, g) is a four-dimensional $\mathcal{W}_1$-manifold, admitting an Abelian complex structure, the Lie algebra $\mathfrak{g}$ of G is given by:*

$$\begin{gathered} C_{13}^1 = C_{14}^2 - C_{12}^4,\ C_{13}^2 = C_{12}^3 - C_{14}^1,\ C_{13}^3 = C_{12}^2 + C_{14}^4,\ C_{13}^4 = -C_{12}^1 - C_{14}^3, \\ C_{24}^1 = -C_{12}^4 - C_{14}^2,\ C_{24}^2 = C_{12}^3 + C_{14}^1,\ C_{24}^3 = C_{12}^2 - C_{14}^4,\ C_{24}^4 = C_{14}^3 - C_{12}^1, \end{gathered} \tag{24}$$

where $C_{ij}^k \in \mathbb{R}$ ($i, j, k = 1, 2, 3, 4$) must satisfy the Jacobi identity (18).

One solution to the equations (18) and (24) is the four-parametric family of Lie algebras $\mathfrak{g}$ defined by

$$\begin{aligned} &[X_1,X_4] = [X_2,X_3] = \lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 + \lambda_4 X_4, \\ &[X_1,X_3] = -[X_2,X_4] = \lambda_2 X_1 - \lambda_1 X_2 + \lambda_4 X_3 - \lambda_3 X_4, \end{aligned} \tag{25}$$

where $\lambda_i \in \mathbb{R}$ ($i = 1, 2, 3, 4$). Thus, by (25) we obtain a four-parametric family of four-dimensional $\mathcal{W}_1$-manifolds with Norden metric.

It has been proved [2] that if a Lie algebra admits an abelian complex structure, then it is solvable. Therefore, the Lie algebras (25) are solvable.

By (20), (22) and (25) we obtain the non-zero components of the Levi-Civita connection of (G, J, g):

$$\begin{aligned}
&\nabla_{X_1} X_1 = \nabla_{X_2} X_2 = \lambda_2 X_3 + \lambda_1 X_4, \quad \nabla_{X_3} X_3 = \nabla_{X_4} X_4 = -\lambda_4 X_1 - \lambda_3 X_2,\\
&\nabla_{X_1} X_3 = \nabla_{X_4} X_2 = \lambda_2 X_1 - \lambda_3 X_4, \quad \nabla_{X_1} X_4 = -\nabla_{X_3} X_2 = \lambda_1 X_1 + \lambda_3 X_3,\\
&\nabla_{X_2} X_4 = \nabla_{X_3} X_1 = \lambda_1 X_2 - \lambda_4 X_3, \quad \nabla_{X_2} X_3 = -\nabla_{X_4} X_1 = \lambda_2 X_2 + \lambda_4 X_4.
\end{aligned} \tag{26}$$

Then, by (19), (20) and (23) we get the following essential non-zero components $F_{ijk} = F(X_i, X_j, X_k)$ of the tensor F:

$$\begin{array}{ll}
\frac{1}{2}F_{222} = F_{112} = F_{314} = \lambda_1, & \frac{1}{2}F_{111} = F_{212} = -F_{414} = \lambda_2,\\
\frac{1}{2}F_{422} = -F_{114} = F_{312} = -\lambda_3, & \frac{1}{2}F_{311} = F_{214} = F_{412} = -\lambda_4.
\end{array} \tag{27}$$

Having in mind (1), (3) and (27), we compute the components $\theta_i = \theta(X_i)$ and $\theta_i^* = \theta^*(X_i)$ of the Lie forms θ and θ^*, respectively:

$$\theta_1 = -\theta_3^* = 4\lambda_2, \quad \theta_2 = -\theta_4^* = 4\lambda_1, \quad \theta_3 = \theta_1^* = 4\lambda_4, \quad \theta_4 = \theta_2^* = 4\lambda_3. \tag{28}$$

A $\mathcal{W}_1$-manifold with closed forms θ and θ^* is called *a conformal Kähler manifold with Norden metric*. The subclass of these manifolds is denoted by $\mathcal{W}_1^0$. Such manifolds are conformally equivalent to Kähler manifolds [1].

We establish that the Lie form θ^* is closed on (G, J, g). Thus, we have

Proposition 2.1. *The manifold (G, J, g) is conformal Kählerian if and only if the Lie form θ is closed, i.e. if and only if one of the conditions holds: $\lambda_1 = \lambda_4$, $\lambda_2 = -\lambda_3$ or $\lambda_1 = -\lambda_4$, $\lambda_2 = \lambda_3$.*

Next, by (2), (10) and (27) we get the square norm of ∇J

$$\|\nabla J\|^2 = 16\left(\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2\right). \tag{29}$$

Proposition 2.2. *The manifold (G, J, g) is isotropic Kählerian if and only if the condition $\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2 = 0$ holds.*

Taking into account (20) and (26), we compute the non-zero components $R_{ijkl} = R(X_i, X_j, X_k, X_l)$ of the curvature tensor R as follows:

$$\begin{array}{lll}
R_{1221} = \lambda_1^2 + \lambda_2^2, & R_{1331} = \lambda_4^2 - \lambda_2^2, & R_{1441} = \lambda_4^2 - \lambda_1^2,\\
R_{2332} = \lambda_3^2 - \lambda_2^2, & R_{2442} = \lambda_3^2 - \lambda_1^2, & R_{3443} = -\lambda_3^2 - \lambda_4^2,\\
R_{1341} = R_{2342} = -\lambda_1\lambda_2, & R_{2132} = -R_{4134} = -\lambda_1\lambda_3, &\\
R_{1231} = -R_{4234} = \lambda_1\lambda_4, & R_{2142} = -R_{3143} = \lambda_2\lambda_3, &\\
R_{1241} = -R_{3243} = -\lambda_2\lambda_4, & R_{3123} = R_{4124} = \lambda_3\lambda_4. &
\end{array} \tag{30}$$

Let us denote $\rho_{ij} = \rho(X_i, X_j)$. Then, by (1), (5) and (30) we obtain the components of the Ricci tensor ρ:

$$\begin{array}{lll} \rho_{11} = 2\big(\lambda_1^2 + \lambda_2^2 - \lambda_4^2\big), & \rho_{12} = -2\lambda_3\lambda_4, & \rho_{23} = 2\lambda_1\lambda_4, \\ \rho_{22} = 2\big(\lambda_1^2 + \lambda_2^2 - \lambda_3^2\big), & \rho_{13} = -2\lambda_1\lambda_3, & \rho_{24} = -2\lambda_2\lambda_4, \\ \rho_{33} = 2\big(\lambda_4^2 + \lambda_3^2 - \lambda_2^2\big), & \rho_{14} = 2\lambda_2\lambda_3, & \rho_{34} = -2\lambda_1\lambda_2, \\ \rho_{44} = 2\big(\lambda_4^2 + \lambda_3^2 - \lambda_1^2\big). & & \end{array} \tag{31}$$

By (26) and (31) we get $\big(\nabla_{X_i}\rho\big)(X_j, X_k) = 0$ for all $i, j, k = 1, 2, 3, 4$.

Proposition 2.3. *The manifold (G, J, g) is Ricci-symmetric.*

Next, by (1), (5) and (31) we obtain the the scalar curvatures as:

$$\tau = 6\big(\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2\big), \qquad \overset{*}{\tau} = -4\big(\lambda_1\lambda_3 + \lambda_2\lambda_4\big). \tag{32}$$

Then, (32), Propositions 2.1 and 2.2 imply

Proposition 2.4. *The considered manifold (G, J, g) has the properties:*

(i) (G, J, g) *is isotropic Kählerian if and only if* $\tau = 0$;
(ii) (G, J, g) *is conformal Kählerian if and only if* $\tau = \overset{*}{\tau} = 0$.

Let us consider the Weyl tensor of (G, J, g). Taking into account (6), (7), (30), (31) and (32), we get $W_{ijkl} = 0$ for all $i, j, k, l = 1, 2, 3, 4$.

Proposition 2.5. *The Weyl tensor of (G, J, g) vanishes. Thus, the curvature tensor has the form $R = \frac{1}{2}\left\{\psi_1(\rho) - \frac{\tau}{3}\pi_1\right\}$.*

By Propositions 2.4 and 2.5 we obtain

Proposition 2.6. *If (G, J, g) is a conformal Kähler manifold, then its curvature tensor has the form $R = \frac{1}{2}\psi_1(\rho)$.*

Further, (7), $\nabla g = 0$, Propositions 2.3 and 2.5 imply

Proposition 2.7. *The manifold (G, J, g) is locally symmetric, i.e. $\nabla R = 0$.*

Let us consider the characteristic two-planes α_{ij} spanned by the basic vectors $\{X_i, X_j\}$ at an arbitrary point of the manifold: totally real two-planes: α_{12}, α_{14}, α_{23}, α_{34} and holomorphic two-planes: α_{13}, α_{24}.

Then, by (7), (8), (20) and (30) we obtain

$$\begin{array}{lll} \nu(\alpha_{12}) = \lambda_1^2 + \lambda_2^2, & \nu(\alpha_{13}) = \lambda_2^2 - \lambda_4^2, & \nu(\alpha_{14}) = \lambda_1^2 - \lambda_4^2, \\ \nu(\alpha_{23}) = \lambda_2^2 - \lambda_3^2, & \nu(\alpha_{24}) = \lambda_1^2 - \lambda_3^2, & \nu(\alpha_{34}) = -\lambda_3^2 - \lambda_4^2. \end{array} \tag{33}$$

Proposition 2.8. *If (G, J, g) is of vanishing holomorphic sectional curvatures, then it is isotropic Kählerian.*

Finally, Propositions 2.2, 2.4 and 2.5 induce

Theorem 2.1. *The following conditions are equivalent:*

(i) (G, J, g) *is isotropic Kählerian*;
(ii) *the condition* $\lambda_1^2 + \lambda_2^2 - \lambda_3^2 - \lambda_4^2 = 0$ *holds*;
(iii) *the scalar curvature* τ *vanishes*;
(iv) *the curvature tensor has the form* $R = \frac{1}{2}\psi_1(\rho)$;

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